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JULY 1990

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SEQUENTIAL ANALYSIS: A METHOD FOR ESTABLISHING RELIABLE RESULTS OF DECK WETNESS PERFORMANCE

P Crossland

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Procurement Executive Ministry of Defence
Haslar GOSPORT Hants PO12 2AG

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By

P Crossland

Summary

This report describes, in some detail, two statistical tests which are designed to be used when carrying out experiments to measure rarely occurring events such as deck wetness. These tests will help to establish more reliable wetness frequency results.

The report describes the theory involved and, by considering two examples of wetness results, shows how the theory is applied.

The mean wetness frequency of the results from each tank can be approximated by a normal distribution, each having its own mean and standard deviation.

The examples in this report have shown that a reduction in run length for experiments to measure deck wetness is possible by using the tests described.

Admiralty Research Establishment
Haslar Gosport Hants PO12 2AG

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Notation

Roman

A	Constant defining the Zone of Inferiority.
B	Constant defining the Zone of Superiority.
C	Constant defining the Critical Region.
E_i	Expected Frequency of Occurrence of Class i.
I_m	Inferiority number after run m.
L	Model Length.
L_T	Tank Length.
m, N_r	Run Number.
N_e	Number of Wave Encounters.
N_{wi}	Number of Wettings on ith run.
N_w	Random Variable (Number of Deck Wettings).
$\bar{N}_w$	Sample Mean Wetness Frequency.
N_{r15}	Number of runs needed to achieve 15 per cent estimated standard error.
$\bar{N}_{w15}$	Average Wetness Frequency after N_{r15} runs.
$\bar{N}_{s5}$	Average Wetness Frequency after Termination of Sequential Test.
O_i	Observed Frequency of Occurrence of Class i.
P_{im}	Sequential Probability Ratio Test.
s	Sample Standard Deviation.
S_m	Superiority Number after Run m.

Greek

α	Probability of Rejecting a Null Hypothesis when it is true.
β	Probability of Accepting a Null Hypothesis when it is false.
μ	True Unknown Mean Deck Wetness Frequency.
μ_o	Estimate of the True Mean Wetness Frequency.
μ_1	Lower Bound for Estimate of Mean Wetness Frequency.
μ_2	Upper Bound for Estimate of Mean Wetness Frequency.
σ	Standard Deviation.

SEQUENTIAL ANALYSIS: A METHOD FOR ESTABLISHING RELIABLE RESULTS OF DECK WETNESS PERFORMANCE

By P Crossland

1. OBJECTIVE

It is current practice to do as many tank runs as is financially viable, when carrying out model experiments to measure rarely occurring events such as deck wetness, slamming, etc in irregular waves. The purpose of this report is to describe some simple statistical tests which can be applied after each tank run to assess whether further runs are required to establish the wetness frequency more reliably.

2. INTRODUCTION

For the purpose of ship design, it is necessary to assess the deck wetness frequency and compare results with different hull forms. Usually strip theory is employed to calculate the rms relative motion at the bow. These calculations have been found to underestimate the deck wetness frequency, probably due to the inability to predict the bow 'swell up'. An alternative is to carry out a series of experiments to measure rare events. However, the dilemma encountered, when doing experiments to measure rare events is in the determination of the run length required to obtain statistics with some specified degree of reliability. It is desirable to carry out a minimum number of tank runs yet still achieve results with a certain degree of reliability.

Crossland and Lloyd (Reference 1) describe experiments carried out at ARE Haslar to measure the deck wetness frequency of the S-175 container ship in an ITTC two parameter wave spectrum. These experiments were part of a collaborative exercise, organised by the 19th ITTC Seakeeping Committee, to determine a more soundly based standard for the run length required for experiments on rarely occurring events (Reference 2).

The results obtained by all participants are listed in Table 1. Taking Participant 3 (ARE Haslar) as an example, the wetness frequency varies between 2 to 14 events per tank run with a mean of 8.89 after 26 runs (See Table 2). The question is whether this is an adequate estimate of the true mean wetness frequency or whether more runs are required to improve the accuracy of the estimate.

The conclusion reached in the ITTC study (Reference 2) was that a run length equivalent to one hour at full scale would give wetness statistics with some degree of reliability for conventional ships at conventional speeds. The results contained a great deal of scatter and in some cases a run length equivalent to one hour was not enough to give reliable statistics. It was decided to develop tests to analyse the results as the experiments progress, and to assess their reliability at each stage of the experiment.

The report describes some of the basic assumptions made about the statistical distribution of the wetness results. The author concludes that, for the purpose of this exercise, the assumption of normality (for the mean wetness frequency) is reasonable.

Two specific statistical tests are described. The first can be applied to situations such as the collaborative experiments, where one model is tested until the results obtained have some statistical reliability. Using this test on the ITTC results, shows that the total run length can be reduced, in some cases.

The second test can be applied to experiments such as those described by Lloyd (Reference 3), where six different above water bow forms were tested to establish the 'optimum bow' for deck wetness. In this case the total required run length has also been reduced.

These tests provide a method of achieving wetness results with a specified degree of accuracy and may lead to a reduction in the number of runs necessary for experiments of this kind.

3. ASSUMPTIONS MADE ABOUT THE DISTRIBUTIONS

A problem that arises frequently in statistical work is in comparing a set of observed and theoretical data. In the case of deck wetness the results may suggest that the data comes from a normal distribution and extensive statistical tests can be carried out on the wetness results such as those given in Table 1 to establish whether or not they can be approximated by normally distributed random variables. However, the tests detailed in this report are statistical tests made on the mean of the wetness data taken from each tank. So, despite the nature of the underlying statistical distribution, it is pertinent to assume normality for the mean of the distribution, given a sample size of at least eight runs.

Table 1 shows the wetness results for all participants and Table 2 gives the mean and standard deviation of the wetness frequency from each of the participants of the collaborative experiments. The results have been left in dimensional form for the purpose of this exercise.

Although it is sufficient to assume that the mean wetness frequency of the results comes from a normal distribution, the results for each tank have their own random distribution and should be treated individually.

4. SINGLE MODEL TEST (TEST 1)

Having assumed that the mean wetness frequencies from each tank are normally distributed random variables (Section 3), it is possible to use the following statistical test.

4.1. Principle of Statistical Analysis

A statistical problem arises when the distribution of a random variable N_w (eg the number of deck wettings per tank run) is unknown and conclusions are to be made about N_w on the basis of a limited number of observations.

However, if the sample size is sufficiently large, ie $N_r \geq 8$, then the mean wetness frequency per tank run, $\bar{N}_w$, can be approximated by the normal distribution and:

$$\bar{N}_w \sim N(\mu, \sigma^2)$$

and the probability of the absolute value of $(N_{wi} - \bar{N}_w)$ greater than a certain value is represented by the shaded area shown in Figure 1(a), where N_{wi} is a value of N_w.

If there were N_r runs carried out as part of an experiment to measure deck wetness frequency, and the number of wettings for each run were recorded as

$$N_{w1}\ ,\ N_{w2}\ ,\ N_{w3}\ ,\ N_{w4}\ ,\ N_{w5}\ ,\ N_{w6}\ \ldots\ldots,\ N_{wNr}$$

Then, μ is estimated by $$N_w = \frac{\Sigma N_{wi}}{N_r} \qquad (1)$$

A statistical test must be applied which determines whether the mean wetness frequency comes from a normal distribution with mean μ_0 or not. This can be represented as two hypothesis, H_0 and H_1 given as

$$H_0 \qquad v \qquad H_1$$

$$\mu = \mu_0 \qquad\qquad \mu \neq \mu_0$$

The test is called a composite hypothesis test.

if $\lvert\bar{N}_w - \mu_0\rvert > C$	reject	H_0
if $\lvert\bar{N}_w - \mu_0\rvert \leq C$	accept	H_0

Where C is a constant depending on the accuracy required (C defines a critical region)

Suppose that the probability of rejecting H_0 when it is true (type I error) is given by α. Then α is known as the significance level of the test (or test size). Size being defined as a measure of the tolerated error and is given by

$$\alpha = P(\ \lvert\ \bar{N}_w - \mu_0\ \rvert\ > C\ /\ \mu = \mu_0) \qquad (2)$$

The above expression reads as 'α equals the probability of $\lvert\bar{N}_w - \mu_0\rvert$ being greater than C given that H_0 is true (ie $\mu = \mu_0$).'

with $$0 \leq \alpha \leq 1$$

Re-write (2) as

$$\alpha = P\left[\frac{|\bar{N}_w - \mu_o|}{\sqrt{(\sigma^2/N_r)}} > \frac{C}{\sqrt{(\sigma^2/N_r)}}\right] \tag{3}$$

By the Central Limit Theorem (Reference 4)

$$\frac{|N_w - \mu_o|}{\sqrt{(\sigma^2/N_r)}} \sim N(0,1)$$

and has the distribution shown in Figure 1(a), with μ and σ set to 0 and 1 respectively. The point at which equation (3) is satisfied is $Z_{\alpha/2}$. These values are tabulated in all statistical tables, such as Reference 4, and a sample of values for $Z_{\alpha/2}$ is given in Table 3.

So, from equation (3), the following expression is derived for C

$$\frac{C}{\sqrt{(\sigma^2/N_r)}} = Z_{\alpha/2} = Z_{1-\alpha/2}$$

By finding a value for C it can be concluded from the original statement that

$$\text{Reject} \quad H_o \quad \text{if} \quad |\bar{N}_w - \mu_o| > Z_{1-\alpha/2}\sqrt{(\sigma^2/N_r)}$$

and

$$\text{Accept} \quad H_o \quad \text{if} \quad |\bar{N}_w - \mu_o| \leq Z_{1-\alpha/2}\sqrt{(\sigma^2/N_r)}$$

In effect bounds have been defined on values of N_w that are acceptable for a test of size α. However, if, as is often the case, the variance must be estimated by the sample variance given by

$$s^2 = \frac{\Sigma\,(N_{wi} - \bar{N}_w)^2}{N_r - 1}$$

then the value of $Z_{1-\alpha/2}$ must be replaced by the Students t-test variable, $t_{N_r-1,1-\alpha/2}$ values for the t-test are given in all statistical tables, such as Reference 4. Typical t-test distributions are shown in Figure 1(b).

Now, the estimated standard error is given by

$$ESE = t_{N_r-1,1-\alpha/2}\sqrt{(s^2/N_r)}$$

The above is for a test of size α.

Alternatively, it could be said that

$$(-t_{N_r-1,1-\alpha/2}\sqrt{(s^2/N_r)},\ t_{N_r-1,1-\alpha/2}\sqrt{(s^2/N_r)})$$

is a $(1-\alpha)$ % confidence limit for $|\bar{N}_w - \mu_o|$.

4.2. Results of Test 1

The test described above was used on a single model to determine when the wetness results become statistically reliable. This analysis should not start until a minimum of, say, 8 tanks runs have been carried out, ensuring that any initial extreme wetness results will not have undesired effects upon the results. Suppose that the required accuracy is 15 per cent, Figure 2 shows the ESE as a function of run number N_r for participant 3. The error decreases as the experiment progresses until the ESE becomes less than the required accuracy level and the experiment can be terminated after 15 runs.

Table 4 shows wetness results for each organisation. It shows the number of runs N_{r15} needed to achieve an ESE of 15 per cent of the running mean.

An accuracy of 15 per cent ESE was chosen because tests of varying levels of accuracy were carried out on the results from each tank and it was found that 15 per cent was a level that was achievable by most tanks. A level of 10 per cent ESE was considered, but only 1 in 3 tanks could achieve this level of accuracy. Also shown in Table 4 is the running mean wetness frequency $\bar{N}_{w15}$ obtained after the 15 per cent accuracy mark had been achieved and the number of runs N_{r15} required to achieve this accuracy.

Analyses, in these cases, have been continued using the remaining data supplied by each tank. The final sample mean $\bar{N}_w$ at the end of the experiment is shown in Table 4 along with the percentage error between the final mean and the running mean calculated at the 15 per cent ESE accuracy level.

In most cases this error is below 10 per cent. This may seem strange given that the ESE is 15 per cent. However, the ESE is an error estimate of the actual mean (ie the average deck wetness after carrying out an infinite number of tank runs) and not an ESE of the sample mean found at the end of the experiment.

5. MULTIPLE MODEL TEST (TEST 2)

The next problem arises when the experimenter tries to compare the merits of two or more bows, when considering deck wetness performance.

Lloyd (Reference 3) carried out a series of experiments to assess the effect of above water hull forms on deck wetness. The experiments involved running a model in irregular head waves, altering its above water bow form, running again and then comparing the two performances. Eighteen runs were carried out for each bow form (equivalent to about one hour full scale for this model). The question is how many runs should be carried out to establish which bow has a superior deck wetness performance. This section describes a method of overcoming this problem. This form of analysis is called sequential analysis.

5.1. Principle of Sequential Analysis

The sequential method of testing a null hypothesis H_o (for example, that Bow A is better than Bow B) may be described as follows.

A rule is given for making one of the following three decisions at any stage of the experiment:

a. To accept the null hypothesis H_o.

b. To reject the null hypothesis H_o.

c. To continue the experiment by making an additional observation.

Such a test is carried out sequentially.

Again, analysis should not start until eight runs have been carried out. So, on the basis of the first eight observations one of the three decisions is made. If the first or second decision is made, the experiment is terminated. If the third decision is made a ninth observation is taken. The process continues on the basis of the first nine observations and so on until the first or second decision is made. There is a more detailed description of the theory of sequential analysis in Reference 5.

A precis of the relevant parts is as follows.

5.2. Testing that the Mean of a Normal Distribution is less than a Given Value

This section deals with the problem of testing the hypothesis that μ is less than or equal to some specified value μ'. This is analogous to determining the most efficient deck wetness performance bow by considering if the mean wetness frequency of a previously tested bow is reliably less than or greater than the mean wetness frequency of the bow currently being tested.

5.2.1. Tolerated Risks of Making Wrong Decisions

If $\mu = \mu'$, the superiority or inferiority of the bow is indeterminate. The preference for judging the bow as superior increases with decreasing value of μ when $\mu < \mu'$, and the preference for judging the bow as inferior increases with increasing value of μ when $\mu > \mu'$. So, it is possible to find two values μ_1 and μ_2 ($\mu_1 < \mu' < \mu_2$): judging a truly superior bow as inferior is considered an error of practical consequence if $\mu \geq \mu_2$; for values of μ between μ_1 and μ_2 the differences in performance are marginal.

Three regions are defined as follows:

a. The zone of superiority consisting of all values of μ for which $\mu \leq \mu_1$

b. The zone of inferiority consisting of all values of μ for which $\mu \geq \mu_2$

c. The zone of indifference consisting of all values of μ between μ_1 and μ_2.

After two values of μ_1 and μ_2 have been chosen the tolerated risk of error can be expressed as:

a. The probability of judging a truly superior bow as inferior should not exceed α.

b. The probability of judging a truly inferior bow as superior should not exceed β.

Thus the tolerated risk of error can be characterised by four numbers μ_1, μ_2, α and β.

The requirements regarding the tolerated risks of making errors are satisfied by the sequential probability ratio test of strength (α, β).

5.2.2. Formulation of the Problem (Sequential Probability Ratio Test)

Suppose the result of an observation is N_{wi}. The value of N_{wi} will vary for each tank run then $\bar{N}_w$, mean of values N_{wi}, has the distribution, (as found in Section 3).

$$\bar{N}_w \sim N(\mu, \sigma^2)$$

with unknown mean μ and known standard deviation σ and it is considered the more desirable the smaller the value of μ. It is possible to designate a particular level μ' such that the bow is judged to be superior if $\mu < \mu_1$.

In the case of deck wetness the plan to test $\mu < \mu_1$ is of most interest.

Suppose that N_{w1}, N_{w2}, ... etc represent the number of wettings observed for tank run 1, 2, ... The probability density of the sample $(N_{w1}, \ldots, N_{wm})$ is given by

$$P_{1m} = \frac{1}{\surd(2\pi)^{m/2}\,\sigma^m} \exp\left(-1/2\sigma^2\, \Sigma(N_{wi} - \mu_1)^2\right) \qquad (4)$$

if $\mu = \mu_1$ and by

$$P_{2m} = \frac{1}{\surd(2\pi)^{m/2}\sigma^m} \exp\left(-1/2\sigma^2 \, \Sigma(N_{wi} - \mu_2)^2\right) \tag{5}$$

if $\mu = \mu_2$. (where $m = N_r$ in these and subsequent equations.)
The ratio P_{2m}/P_{1m} is computed at each stage of the analysis (ie after each tank run).

Then, there exist constants A and B (dependant on the tolerated risk of error). So that additional runs are made as long as

$$B < P_{2m}/P_{1m} < A \tag{6}$$

Then the experiment is terminated and the bow is judged to be superior if

$$P_{2m}/P_{1m} \leq B \tag{7}$$

and the experiment is terminated and the bow is judged to be inferior if

$$P_{2m}/P_{1m} \geq A \tag{8}$$

In reality A and B are indeterminate, but the following approximations are made

Take $A = (1 - \beta)/\alpha$

and $B = \beta/(1 - \alpha)$

For α and β defined earlier.

Now, from (4) and (5)

$$\frac{P_{2m}}{P_{1m}} = \frac{\exp(-1/\sigma^2 \, \Sigma(N_{wi} - \mu_2)^2)}{\exp(-1/\sigma^2 \, \Sigma(N_{wi} - \mu_1)^2)} \tag{9}$$

By simplifying (9), taking the natural logarithms of (6) .. (9) and rearranging, the inequalities (6) ... (8) become,

$$\frac{\sigma^2}{\mu_2 - \mu_1}\log_e(\beta/(1-\alpha)) + \frac{m}{2}(\mu_2 + \mu_1) < \Sigma N_{wi} < \frac{\sigma^2}{\mu_2 - \mu_1}\log_e((1-\beta)/\alpha) + \frac{m}{2}(\mu_2 + \mu_1)$$

$$\Sigma N_{wi} \leq \frac{\sigma^2}{\mu_2 - \mu_1}\log_e(\beta/(1 - \alpha)) + \frac{m}{2}(\mu_2 + \mu_1)$$

$$\Sigma N_{wi} \geq \frac{\sigma^2}{\mu_2 - \mu_1} \log_e((1 - \beta)/a) + \frac{m}{2} (\mu_2 + \mu_1)$$

respectively.

The inspection plan will be as follows.

For each run m compute the superiority number.

$$S_m = \frac{\sigma^2}{\mu_2 - \mu_1} \log_e(\beta/1(1 - \alpha)) + \frac{m}{2} (\mu_2 + \mu_1)$$

and the inferiority number

$$I_m = \frac{\sigma^2}{\mu_2 - \mu_1} \log_e((1 - \beta)/\alpha) + \frac{m}{2} (\mu_2 + \mu_1)$$

The experiment will continue so long as

$$S_m < \Sigma N_{wi} < I_m \qquad (10)$$

The experiment will terminate as soon as the inequality (10) is not satisfied, the bow will be judged to be superior if

$$\Sigma N_{wi} \leq S_m$$

and the bow will be judged to be inferior if

$$\Sigma N_{wi} \geq I_m$$

For the case when the variance σ^2 is unknown (as is the case here), the sample variance s^2 is used and must be recalculated after each tank run.

5.2.3. Choice of Values for μ_1, μ_2, α and β

As mentioned in Section 5.2.1, μ_1, μ_2, α and β characterise the tolerated risks and their values must be known before analysis can commence. μ_1 and μ_2 define the zones of superiority and inferiority respectively and so represent the points at which a change in mean wetness (due to bow shape for example) is regarded as being significant. Values of ± 10 per cent of the test mean μ have been chosen for μ_1 and μ_2 respectively. So,

$$\mu_1 = 0.9\mu, \qquad \mu_2 = 1.1\mu$$

α and β represent the probability of committing two types of errors, and so these have been chosen to be

$$\alpha = 0.05$$

$$\beta = 0.05$$

Since 5 per cent probability of making an error is regarded as reasonable.

5.3. Results of Test 2

Lloyd's actual wetness data have not been tabulated in any technical report and are shown in Table 5. Also shown in Table 5 is the final ordering of bow performance as concluded by Lloyd. These data have been used to test the theory described in the previous sections.

The test procedure is as follows:

a. Choose any bow to begin with: for this example the chosen bow is Bow 45.

b. Carry out 8 tank runs, then start analysis using Test 1.

Continue to add to the wetness results until the ESE falls below 15 per cent. Figure 3 shows the ESE as a function of run number for Bow 45. The probable error again decreases as the experiment progresses. However, Lloyd only carried out 18 runs on this particular bow form and so the experiment terminates even though the required accuracy level is not reached. The sample mean obtained at this stage ($\bar{N}_{s5}$ = 3.94) is recorded and shown in Table 6. Let this be known as trial 1.

c. Use Test 2.

Once reliable statistics have been obtained for Bow 45 using Test 1, the next stage is the comparative test. Choose another bow to test against the mean wetness frequency obtained from Bow 45, say Bow 50. Call this comparison Trial 2, Figure 4 shows the total number of wettings observed for Bow 50 as a function of run number. Also shown are the zones of superiority, inferiority and indifference. The wetness line remains within the zone of indifference until 13th run, after which it can be concluded that Bow 50 is inferior to Bow 45. So, it is now possible to order the bows in terms of deck wetness performance as shown in Table 6. Continue the process, by testing another bow, say Bow 40. Call it trial 3 and test it against the mean wetness frequency of Bow 45. Figure 5 shows the total number of wettings observed for Bow 40 as a function of run number, along with the three zones mentioned above. Here, the process does not terminate until all the available data have been used, so no conclusion about which bow is superior can be drawn because at the termination stage the sum number of wettings still lies within the zone of indifference. This in effect is what might happen in an experiment situation, that is, a conclusion must be drawn about the relative performance of the two bows after a certain number of runs, and this conclusion may well be that the performance of the bows is essentially similar.

The order of deck wetness performance can be updated at this stage and is shown as the result of trial 3 in Table 6. Bow 40 has the same performance as Bow 45 and the average of the mean wetness frequencies subsequently becomes the test mean. This process continues until all the bows have been tested.

6. CONCLUSIONS

This report has detailed a method of analysing wetness statistics as the experiments progress, in an attempt to provide results that are statistically reliable and to reduce the number of tank runs. Because of the assumptions made, these methods should only be used as a rough guide to rejecting inferior ship designs. However, the method, when applied to the available data, produces results that are believable.

Table 4 shows the run lengths needed, for each participant of the ITTC experiments, to achieve 15 per cent ESE. Now, considering only those tanks who achieved the specified wave condition, the average run length needed to achieve reliable results in about 174 model lengths. The recommendation given in Reference 1 (based on subjective estimates of scatter) is for a run of 200 model lengths to give statistically reliable results, but the analyses described in this memorandum confirms that the run length could be reduced to 170 model lengths.

A test has been demonstrated which assists in assessing the order of superiority of modifications to hull forms aimed at reducing deck wetness. The method has been applied to Lloyd's series of experiments on bow forms. It should be noted that the test only determines the superior bow and perhaps the second most superior. Subsequent inferior bows are ordered according to their mean wetness frequencies. It is possible, if required, to use test 2 extensively to determine the correct ordering of bows and to assess the reliability of that order. For example, testing Bow 35 against Bow 40 to determine which is superior etc.

It should be noted that the mean wetness frequencies, in this table are calculated after the end of the sequential test.

The amount of runs saved will depend upon, most of all, the parameters used to define the tolerated risks. (ie μ_1, μ_2, α and β). It will also depend upon the order in which the bows are chosen because the trial will terminate more quickly if the difference in the mean deck wetness frequency is large. So, this may determine the run order when carrying out experiments of this type.

Simple computer programs can be written to analyse the results as the experiment progresses thus assisting an experimenter in making quantitative decisions.

The report detailed a method of analysing wetness results to obtain reliable statistics. However, there is an inherent assumption that the mean wetness frequencies from each tank are normally distributed random variables. This assumption is valid for higher values of $\bar{N}_w$. For lower values the distribution becomes highly skewed since, the minimum mean wetness frequency that can exist is zero. This is illustrated in Figure 6. So, for deck wetness experiments, a sufficiently severe wave condition must be chosen to establish a wetness frequency exceeding, say, 5 wettings per tank run. However, the wave condition must not be so severe that a wetting occurs at every wave encounter. In this case the number of wettings cannot exceed the number of waves encountered N_e and again the distribution will become highly skewed.

7. RECOMMENDATIONS

The methods described in this report are designed to aid the experimenter in assessing the relative merits of different ship designs when carrying out experiments to measure deck wetness. The final decision, to reject a design or not, inevitably lies with the experimenter. Whilst these tests give the minimum requirements for reliable results, the experimenter should strive to carry out as many runs as possible, within the necessary constraints of time and cost, to obtain results that are as reliable as possible.

8. ACKNOWLEDGEMENT

The author acknowledges Mr Bernard Higgins of Portsmouth Polytechnic for his critical views and help in this study.

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Table 1

WETNESS RESULTS OBTAINED AT FP (ALL PARTICIPANTS)

Run N_r.	Participant Number of wettings N_w in each tank run														
	1	2	3	4	5*			6	7	8	9	10	11	12	13
1	4	3	10	19	3	7	5	9	10	10	9	12	37	10	9
2	3	4	12	17	6	6	6	3	11	8	7	14	34	12	4
3	1	2	11	13	4	1	0	10	11	7	8	14	24	9	4
4	2	3	6	15	5	5	3	7	9	8	9	12	25	11	3
5	1	4	12	14	6	3	3	13	10	7	9	13	23	12	6
6	3	3	13	11	1	8	8	10	11	7	6	9	26	11	5
7	4	2	14	15	2	4	1	15	11	10	5	13	25	12	7
8	0	2	8	14	1	6	5	10	9	7	3	13	24	12	8
9	2	3	8	11	3	3	2	9	11	6	8	11	24	10	8
10	3	3	13	12	1	2	3	11	7	6	8	12	22	11	7
11	2	4	7	12	3	3	2	10	10	4	6	7			8
12	4	4	6	16	4	6	10	11	6	5	4	15			8
13	4	2	10	14	4	6	5	8	8	8	9	14			4
14	3	1	9	13	2	5	3	12	8	8	8	7			6
15	4	2	8	16	5	3	1	7	9	8	7	8			12
16	5	4	4	18	5	3	1	7	9	5	7	10			6
17	3	2	8	14	3	3	2	13	11	7	9	9			10
18	2	2	11	14	4	3	1	8	11	9	6	11			10
19	2	3	2	15	4	3	1	7	5	8	7	8			6
20	2	4	8	15	3	1	2	8	10	7	10	13			8
21	1	4	4	15	3	2			4	7	7				
22	4	1	7		6	4			9	5	10				
23	3	3	13		3	3			8	8	8				
24	3		8		3				8	5	8				
25	1		5		3				9	7	7				
26	4		14		10					5					
27	3									5					
28	4									5					
29	2									6					
30	2									4					
31	4									7					
32	4									5					
33	4									5					
34	2									9					
35	2									5					
36	0									6					
37	2									12					
38	2									6					
39	2														
40	2														
41	2														
42	2														

* Results from three different Froude numbers Fn = 0.17, 0.21, 0.25 respectively

Table 2

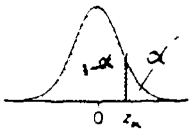
SAMPLE MEAN AND STANDARD DEVIATION OF WETNESS RESULTS

Organisation	Sample Mean $\bar{N}_w$	Sample Standard Deviation s
1	2.60	1.19
2	2.83	0.95
3	8.89	3.30
4	14.43	2.09
5*	3.73	1.93
6	9.80	2.32
7	9.00	1.94
8	14.32	1.80
9	7.40	1.73
10	11.25	2.51
11	21.40	9.85
12	11.00	1.02
13	6.95	2.33

Key: * Result for Fn = 0.25

Table 3

VALUES FOR Z_α



α	.000	.001	.002	.003	.004	.005	.006	.007	.008	.009	.010	
.00	∞	3.0902	2.8782	2.7478	2.6521	2.5758	2.5121	2.4573	2.4089	2.3656	2.3263	.99
.01	2.3263	2.2904	2.2571	2.2262	2.1973	2.1701	2.1444	2.1201	2.0969	2.0749	2.0537	.98
.02	2.0537	2.0335	2.0141	1.9954	1.9774	1.9600	1.9431	1.9268	1.9110	1.8957	1.8808	.97
.03	1.8808	1.8663	1.8522	1.8384	1.8250	1.8119	1.7991	1.7866	1.7744	1.7624	1.7507	.96
.04	1.7507	1.7392	1.7279	1.7169	1.7060	1.6954	1.6849	1.6747	1.6646	1.6546	1.6449	.95
.05	1.6449	1.6352	1.6258	1.6164	1.6072	1.5982	1.5893	1.5805	1.5718	1.5632	1.5548	.94
.06	1.5548	1.5464	1.5382	1.5301	1.5220	1.5141	1.5063	1.4985	1.4909	1.4833	1.4758	.93
.07	1.4758	1.4684	1.4611	1.4538	1.4466	1.4395	1.4325	1.4255	1.4187	1.4118	1.4051	.92
.08	1.4051	1.3984	1.3917	1.3852	1.3787	1.3722	1.3658	1.3595	1.3532	1.3469	1.3408	.91
.09	1.3408	1.3346	1.3285	1.3225	1.3165	1.3106	1.3047	1.2988	1.2930	1.2873	1.2816	.90
.10	1.2816	1.2759	1.2702	1.2646	1.2591	1.2536	1.2481	1.2426	1.2372	1.2319	1.2265	.89
.11	1.2265	1.2212	1.2160	1.2107	1.2055	1.2004	1.1952	1.1901	1.1850	1.1800	1.1750	.88
.12	1.1750	1.1700	1.1650	1.1601	1.1552	1.1503	1.1455	1.1407	1.1359	1.1311	1.1264	.87
.13	1.1264	1.1217	1.1170	1.1123	1.1077	1.1031	1.0985	1.0939	1.0893	1.0848	1.0803	.86
.14	1.0803	1.0758	1.0714	1.0669	1.0625	1.0581	1.0537	1.0494	1.0450	1.0407	1.0364	.85
.15	1.0364	1.0322	1.0279	1.0237	1.0194	1.0152	1.0110	1.0069	1.0027	0.9986	0.9945	.84
.16	0.9945	0.9904	0.9863	0.9822	0.9782	0.9741	0.9701	0.9661	0.9621	0.9581	0.9542	.83
.17	0.9542	0.9502	0.9463	0.9424	0.9385	0.9346	0.9307	0.9269	0.9230	0.9192	0.9154	.82
.18	0.9154	0.9116	0.9078	0.9040	0.9002	0.8965	0.8927	0.8890	0.8853	0.8816	0.8779	.81
.19	0.8779	0.8742	0.8705	0.8669	0.8633	0.8596	0.8560	0.8524	0.8488	0.8452	0.8416	.80
.20	0.8416	0.8381	0.8345	0.8310	0.8274	0.8239	0.8204	0.8169	0.8134	0.8099	0.8064	.79
.21	0.8064	0.8030	0.7995	0.7961	0.7926	0.7892	0.7858	0.7824	0.7790	0.7756	0.7722	.78
.22	0.7722	0.7688	0.7655	0.7621	0.7588	0.7554	0.7521	0.7488	0.7454	0.7421	0.7388	.77
.23	0.7388	0.7356	0.7323	0.7290	0.7257	0.7225	0.7192	0.7160	0.7128	0.7095	0.7063	.76
.24	0.7063	0.7031	0.6999	0.6967	0.6935	0.6903	0.6871	0.6840	0.6808	0.6776	0.6745	.75
.25	0.6745	0.6713	0.6682	0.6651	0.6620	0.6588	0.6557	0.6525	0.6495	0.6464	0.6433	.74
.26	0.6433	0.6403	0.6372	0.6341	0.6311	0.6280	0.6250	0.6219	0.6189	0.6158	0.6128	.73
.27	0.6128	0.6098	0.6068	0.6038	0.6008	0.5978	0.5948	0.5918	0.5888	0.5858	0.5828	.72
.28	0.5828	0.5799	0.5769	0.5740	0.5710	0.5681	0.5651	0.5622	0.5592	0.5563	0.5534	.71
.29	0.5534	0.5505	0.5476	0.5446	0.5417	0.5388	0.5359	0.5330	0.5302	0.5273	0.5244	.70
.30	0.5244	0.5215	0.5187	0.5158	0.5129	0.5101	0.5072	0.5044	0.5015	0.4987	0.4959	.69
.31	0.4959	0.4930	0.4902	0.4874	0.4845	0.4817	0.4789	0.4761	0.4733	0.4705	0.4677	.68
.32	0.4677	0.4649	0.4621	0.4593	0.4565	0.4538	0.4510	0.4482	0.4454	0.4427	0.4399	.67
.33	0.4399	0.4372	0.4344	0.4316	0.4289	0.4261	0.4234	0.4207	0.4179	0.4152	0.4125	.66
.34	0.4125	0.4097	0.4070	0.4043	0.4016	0.3989	0.3961	0.3934	0.3907	0.3880	0.3853	.65
.35	0.3853	0.3826	0.3799	0.3772	0.3745	0.3719	0.3692	0.3665	0.3638	0.3611	0.3585	.64
.36	0.3585	0.3558	0.3531	0.3505	0.3478	0.3451	0.3425	0.3398	0.3372	0.3345	0.3319	.63
.37	0.3319	0.3292	0.3266	0.3239	0.3213	0.3186	0.3160	0.3134	0.3107	0.3081	0.3055	.62
.38	0.3055	0.3029	0.3002	0.2976	0.2950	0.2924	0.2898	0.2871	0.2845	0.2819	0.2793	.61
.39	0.2793	0.2767	0.2741	0.2715	0.2689	0.2663	0.2637	0.2611	0.2585	0.2559	0.2533	.60
.40	0.2533	0.2508	0.2482	0.2456	0.2430	0.2404	0.2378	0.2353	0.2327	0.2301	0.2275	.59
.41	0.2275	0.2250	0.2224	0.2198	0.2173	0.2147	0.2121	0.2096	0.2070	0.2045	0.2019	.58
.42	0.2019	0.1993	0.1968	0.1942	0.1917	0.1891	0.1866	0.1840	0.1815	0.1789	0.1764	.57
.43	0.1764	0.1738	0.1713	0.1687	0.1662	0.1637	0.1611	0.1586	0.1560	0.1535	0.1510	.56
.44	0.1510	0.1484	0.1459	0.1434	0.1408	0.1383	0.1358	0.1332	0.1307	0.1282	0.1257	.55
.45	0.1257	0.1231	0.1206	0.1181	0.1156	0.1130	0.1105	0.1080	0.1055	0.1030	0.1004	.54
.46	0.1004	0.0979	0.0954	0.0929	0.0904	0.0878	0.0853	0.0828	0.0803	0.0778	0.0753	.53
.47	0.0753	0.0728	0.0702	0.0677	0.0652	0.0627	0.0602	0.0577	0.0552	0.0527	0.0502	.52
.48	0.0502	0.0476	0.0451	0.0426	0.0401	0.0376	0.0351	0.0326	0.0301	0.0276	0.0251	.51
.49	0.0251	0.0226	0.0201	0.0175	0.0150	0.0125	0.0100	0.0075	0.0050	0.0025	0.0000	.50
	.010	.009	.008	.007	.006	.005	.004	.003	.002	.001	.000	1-α

Table 4

SINGLE MODEL TEST

Tank		$\frac{L_T}{L}$	N_r	$\bar{N}_w$	N_{r15}	$\bar{N}_{w15}$	$\frac{N_{r15}L_T}{L}$	% Error
1		7.6	42	2.60	34	2.79	258	8
2		29.7	23	2.83	21	2.91	624	3
3	*	18.4	26	8.89	15	9.80	276	10
4	*	21.4	21	14.43	8	14.75	171	2
5		Did not achieve desired accuracy level						
6	*	15.5	20	9.80	11	10.55	171	8
7	*	22.0	25	9.00	8	10.00	176	11
8		13.0	38	14.32	8	15.63	104	9
9		9.6	25	7.40	15	7.07	144	5
10	*	18.4	20	11.25	8	12.50	147	11
11		27.4	10	21.38	8	27.25	219	27
12	*	13.1	10	11.00	8	11.13	105	1
13	*	20.4	20	6.95	20	6.95	408	0
Mean run length of valid tests							174	

Key: * Achieved wave condition specified by the ITTC Committee for the purpose of the collaborative experiments.

Table 5

RESULTS OF LLOYDS WETNESS EXPERIMENTS

Run	Number of Wettings per Run					
Bow	30	35	40	45	50	55
WG120	4	4	3	3	5	2
158	4	4	6	6	5	4
194	5	4	4	4	4	2
229	2	6	5	5	7	6
265	5	5	5	5	7	4
301	2	3	2	2	3	1
WH120	0	2	2	2	4	3
172	2	3	5	5	2	3
207	2	2	4	4	2	3
242	5	–	5	5	6	5
277	3	4	3	4	4	1
312	2	6	5	5	5	4
WV120	3	3	4	4	6	4
174	2	2	2	2	6	1
209	1	1	2	2	3	1
244	3	3	4	4	5	4
279	3	4	5	5	4	3
314	4	4	4	4	8	4
Mean	2.89	3.53	3.89	3.94	4.78	3.11
Order	1	3	4	5	6	2

Table 6

MULTIPLE MODEL TEST

Bow	45	50	40	55	35	30
N_{s5}	3.94	4.69	3.89	2.90	3.88	2.89
Runs	18	13	18	10	8	18
Trial 1	1					
2	1	2				
3	1	3	1			
4	3	4	2	1		
5	4	5	3	1	2	
Final Order	5	6	4	2	3	1

Total number of runs 82

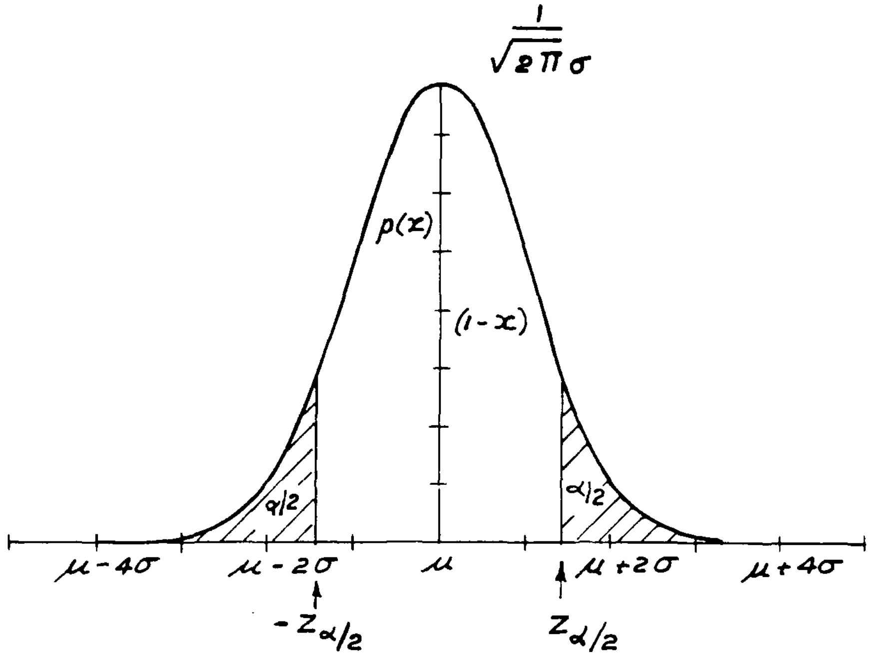


FIG. 1(a). NORMAL DISTRIBUTION

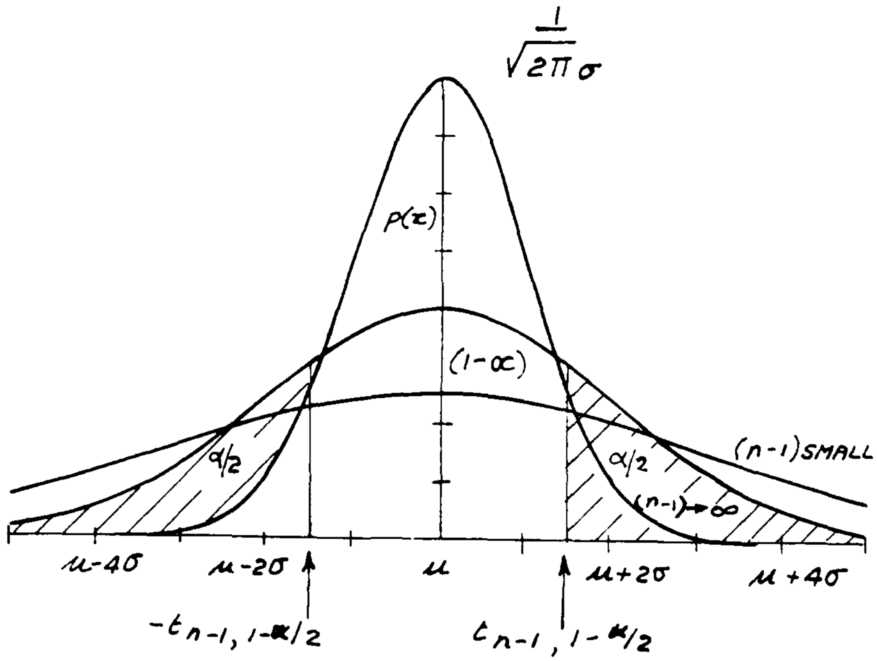


FIG. 1(b) STUDENTS t - DISTRIBUTIONS

FIG.1. STATISTICAL DISTRIBUTIONS

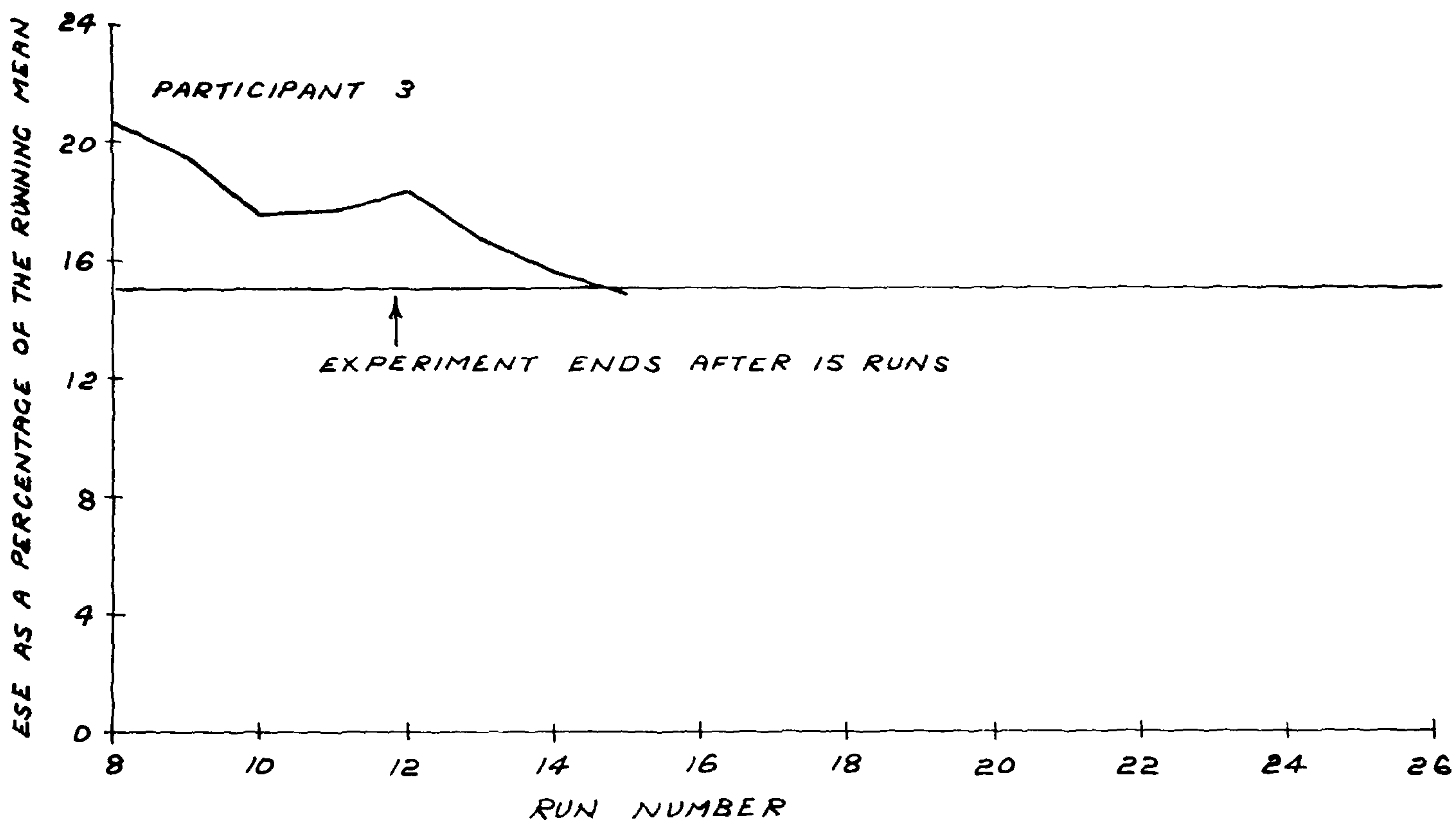


FIG. 2. DEVELOPMENT OF THE ESTIMATED STANDARD ERROR AS THE EXPERIMENT PROGRESSES

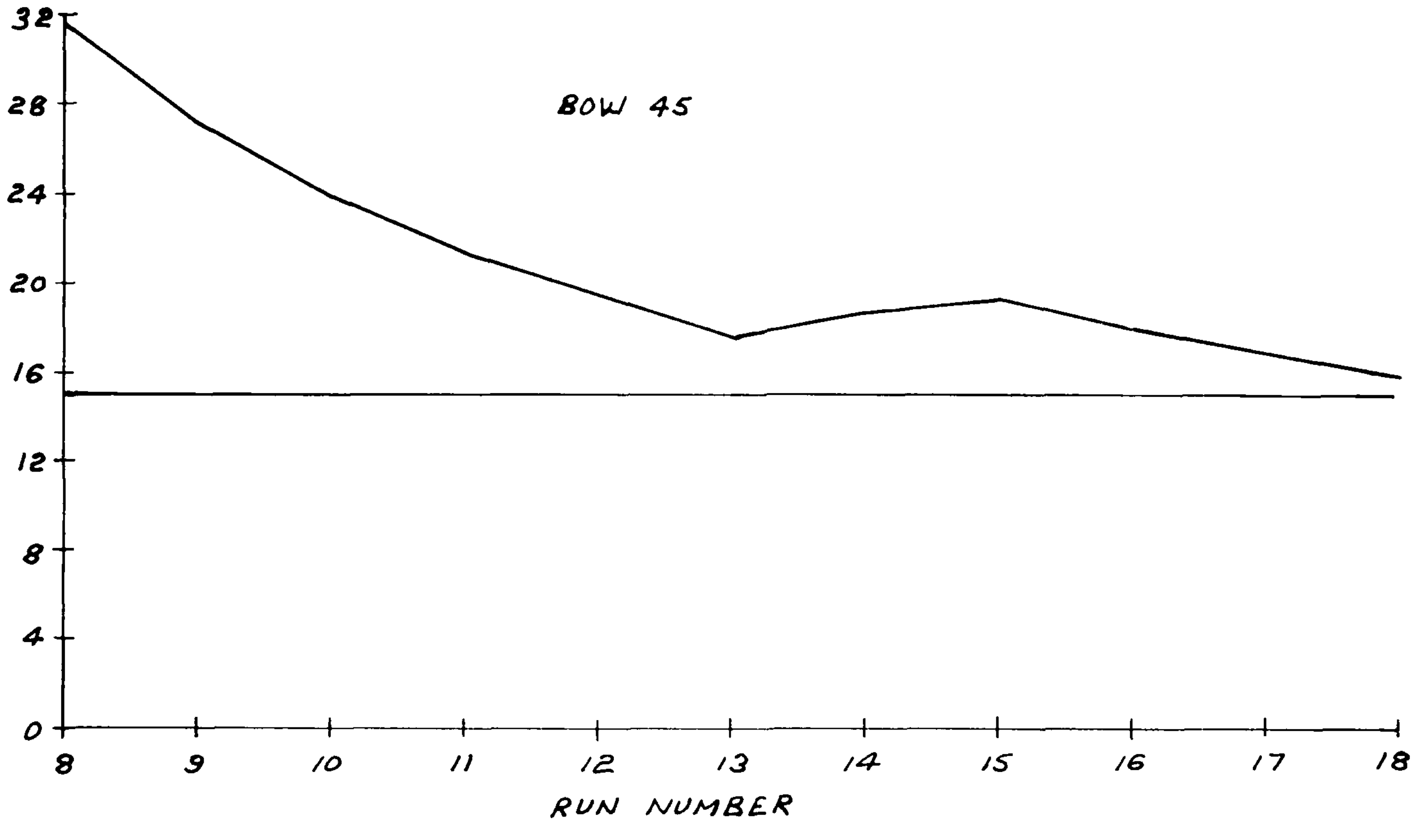


FIG. 3. DEVELOPMENT OF THE ESTIMATED STANDARD ERROR AS THE EXPERIMENT PROGRESSES

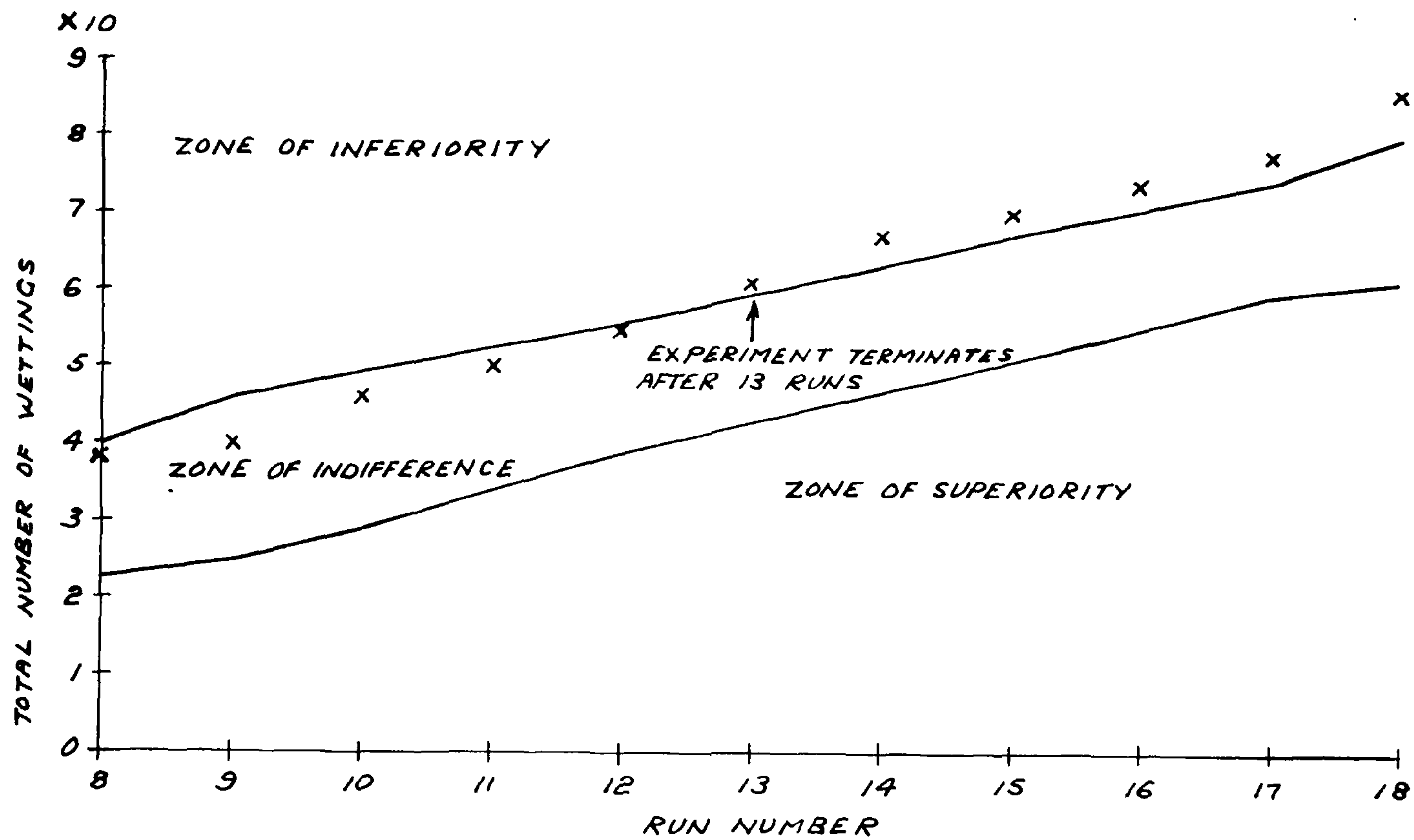


FIG. 4. RESULTS OF COMPARING BOW 50 WITH THE MEAN WETNESS FREQUENCY OF BOW 45

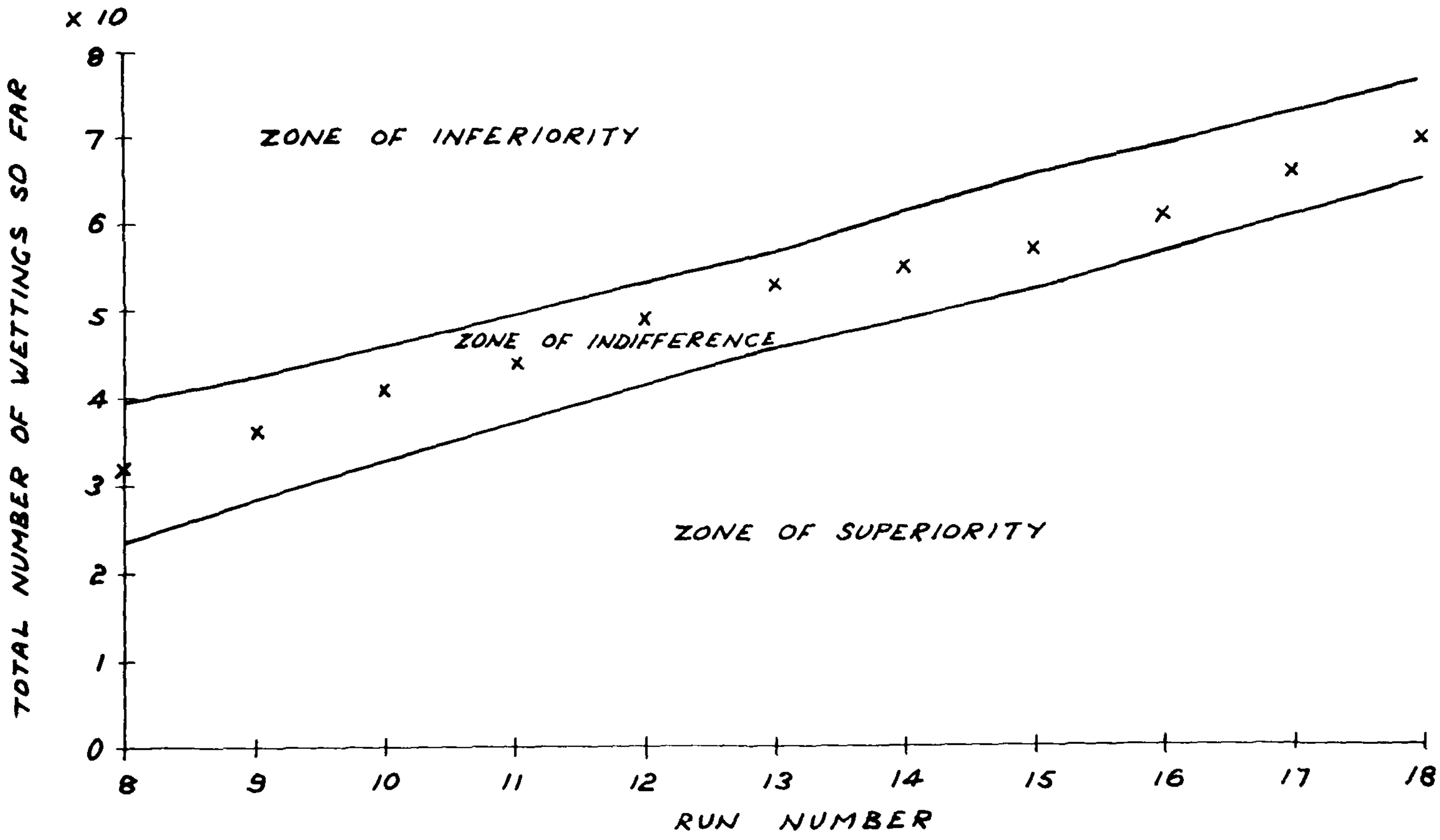


FIG. 5. RESULTS OF COMPARING BOW 40 WITH THE MEAN WETNESS FREQUENCY OF BOW 45

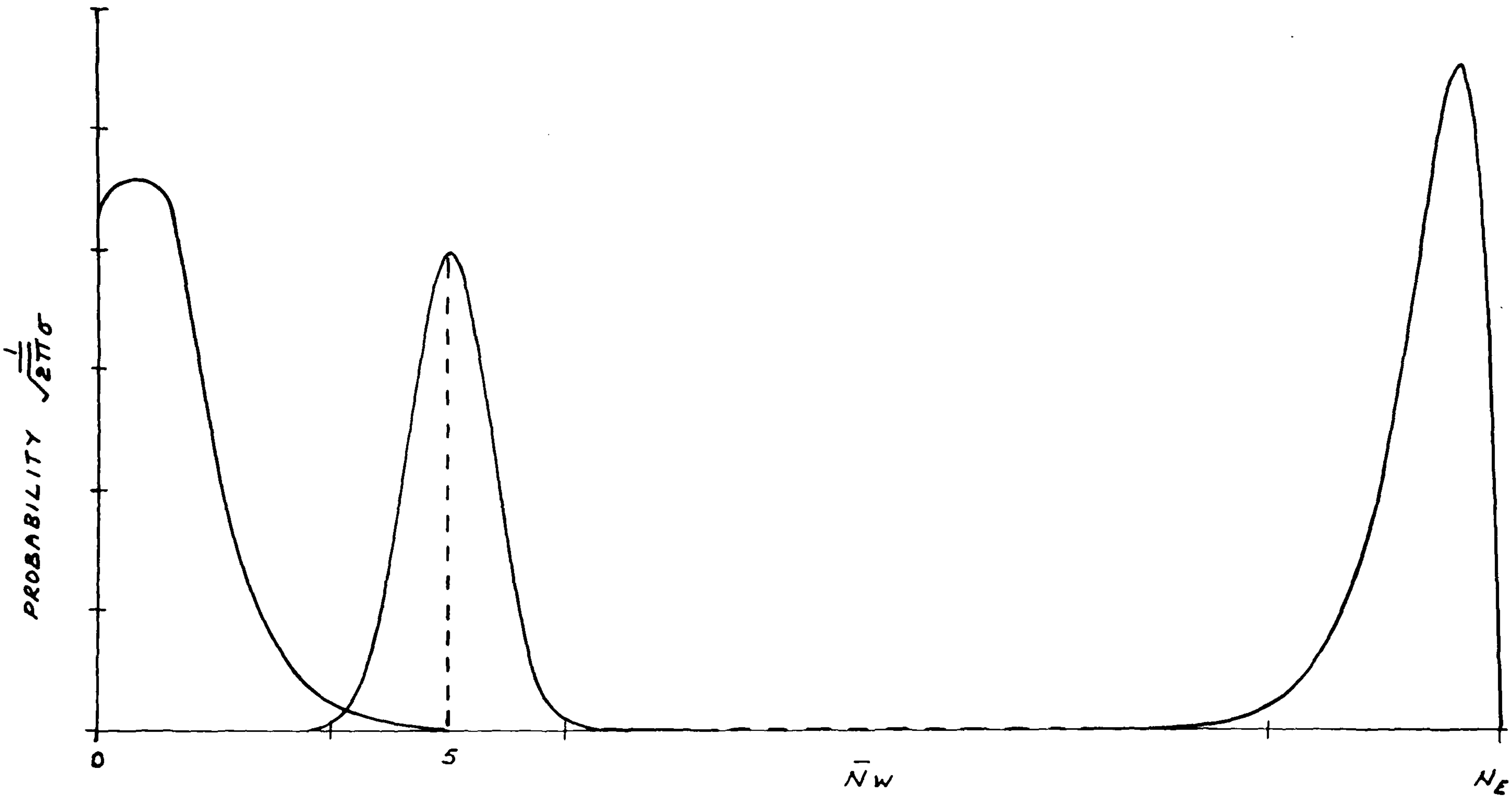


FIG. 6. SKEWED AND UNSKEWED STATISTICAL DISTRIBUTIONS

REPORT DOCUMENTATION PAGE DRIC Reference number (if known)

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Originator's Reference/Report No	Month	Year
ARE TM(UHR)90309	JULY	1990

Originator's Name and Location

ADMIRALTY RESEARCH ESTABLISHMENT
HASLAR GOSPORT HANTS PO12 2AG

Monitoring Agency Name and Location

Title

SEQUENTIAL ANALYSIS: A METHOD FOR ESTABLISHING RELIABLE RESULTS OF DECK WETNESS

Report Security Classification	Title Classification (U, R, C or S)
UNLIMITED	U

Foreign Language Title (In the case of translations)

Conference Details

Agency Reference	Contract Number and Period

Project Number	Other References

Authors	Pagination and Ref
P CROSSLAND	

Abstract This report describes, in some detail, two statistical tests which are designed to be used when carrying out experiments to measure rarely occurring events such as deck wetness. These tests will help to establish more reliable wetness frequency results.
The report describes the theory involved and, by considering two examples of wetness results, shows how the theory is applied.
The mean wetness frequency of the results from each tank can be approximated by a normal distribution, each having its own mean and standard deviation.
The examples in this report have shown that a reduction in run length for experiments to measure deck wetness is possible by using the tests described.

Abstract Classification (U, R, C or S)

Descriptors

SHIP TESTING, STATISTICAL ANALYSIS

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